

Integrated Aircraft–Controller Design Using Linear Matrix Inequalities

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A new methodology is presented for solving the problem of optimizing the aircraft parameters, such as the size of its control surfaces, while meeting open- and closed-loop static and dynamic performance requirements. The approach proposed involves rewriting these requirements as linear matrix inequalities and solving a related constrained optimization problem for which efficient numerical solutions are available.

I. Introduction

ALTHOUGH significant progress has been made in the area of integrated optimal structure–controller design (e.g., Refs. 1–3 and references therein), little has been done to apply control theory to integrated aircraft–controller design. However, significant improvements in the overall aircraft performance and cost are possible if the process of control-systems development were to be integrated with aircraft design itself. In this paper, we develop a new methodology that provides a numerical framework for the integrated aircraft–controller design.

The idea of applying control theory to aircraft design has been explored by Jameson and Reuther in a number of papers (Refs. 4 and 5 and references therein). In particular, Jameson⁴ considers the problem of optimal airfoil design. This problem is formulated as an optimal control problem for systems described by partial differential equations. The solution is a two-dimensional optimal airfoil shape described by a conformal mapping from a unit circle. These results were extended by Reuther and Jameson⁵ to include three-dimensional airfoil profiles.

Rather than using control theory for aircraft design, this paper considers a problem of combining the design of some aircraft parameters with the control-system development. The parameters to be synthesized include aircraft control-surface sizes. Furthermore, the control system obtained as a result of the integrated design process is expected to meet the flying-quality requirements for the new configuration of the aircraft control-surface sizes.

Traditionally, the complete aircraft–control system design process includes the following steps:

- 1) Designing the wing–body (baseline) configuration to meet the mission requirements such as payload, range, powerplant, and maximum speed.
- 2) Designing and appending control surfaces to the baseline vehicle to provide sufficient control power for controllability, stability, and dynamic performance.
- 3) Designing the control system for the complete aircraft for improved handling qualities and dynamic performance.

The sequential nature of this process (i.e., in practice, step 3 always follows step 2) results in aircraft control surfaces that are usually oversized. Unfortunately, this comes at a great expense with regard to weight, drag, stealth, and financial cost. Consequently, it is desirable to incorporate only the amount of control power that is necessary to attain the desired dynamic performance requirements. This issue has become particularly important since the introduction of reduced static stability (RSS) aircraft has made control-surface sizing a critical element of the aircraft control-power design process. On the one hand, inadequate control power is directly responsible for the loss of numerous aircraft. To oversize the controls, however,

is to forfeit the benefits of RSS, resulting in poorer performance, increased weight, increased cost, increased drag, and decreased stealth.

The problem of integrating control-power and control-systems design process for RSS aircraft has been addressed in a number of studies sponsored by the U.S. Air Force and NASA. Lapin et al.⁶ developed algebraic closed-form expressions for the gains of a fixed-structure proportional plus integral controller for longitudinal and lateral control of an aft-swept close-coupled canard RSS aircraft. The expressions derived were functions of aircraft characteristics, flight conditions, and desired handling qualities and were used in determining the control-power and control-system requirements.

This paper extends the work of Lapin et al.⁶ by introducing an entirely different approach to the problem of integrated aircraft control power–controller design. The key idea is to formulate a constrained optimization problem (COP), where the cost J to be minimized is a linear function of the weighted aircraft control-surface parameters; the search is done over the set of feedback controllers that meet the flying-quality and maneuverability requirements. The cost function J may include weights such as cost per unit mass of the control surfaces, weight per unit mass, stealth penalty per unit area, and so forth. In this paper, we show that many flying qualities as well as maneuverability requirements considered by Lapin et al.⁶ and many standard MIL-F-8785 requirements⁷ can be expressed as linear matrix inequalities (LMIs). The main contribution of the paper is that no a priori assumption is made on controller structure and the order of the aircraft model. Thus, the actuator and higher-order dynamics can be included in the problem formulation directly without simplifying assumptions. Moreover, the search is done over a large set of controllers that satisfy the design requirements. Unconventional control effectors such as directed thrust can be incorporated easily in the proposed methodology. Furthermore, the proposed formulation allows the designer to determine the cost function best suited for the problem at hand. For example, if the main concern is the monetary cost of adding control surfaces to the basic aircraft, then the cost-function weights can be selected to represent cost per unit mass of the control surfaces. Finally, additional constraints such as stability robustness in the presence of structured and unstructured uncertainties not considered by Lapin et al.⁶ can be incorporated easily in the problem formulation.

LMIs have been used in systems and control for over a century. The first LMI can be attributed to Lyapunov, who showed that the asymptotic stability of the differential equation $\dot{x} = Ax$ is related to the inequality $A^T P + PA < 0$ having a positive definite solution: $P > 0$. The expressions $P > 0$ and $A^T P + PA < 0$ are LMIs. Since then, LMIs have been used widely to solve control-systems analysis and design problems. An interesting historical perspective on LMIs in systems and control can be found in Ref. 8. More recently, it has been shown that such well-known control problems as \mathcal{H}_2 and \mathcal{H}_∞ synthesis also can be formulated as LMIs.

In this paper, we show that many aircraft dynamic and static performance requirements can be rewritten as LMIs. The main attraction of using LMIs to express flying qualities, maneuverability, and

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other requirements is the availability of numerically efficient computational methods to solve them.^{8–10} In particular, Gahinet and Nemirovskii^{9,10} recently released a preliminary version of LMI-Lab, a MATLABTM toolbox for solving LMIs. This software was used to obtain numerical solutions for the integrated plant–controller problem proposed in this paper. In particular, we show that performance specifications that are posed as LMIs provide a means not only for solving the controller design problem, but also for reducing the necessary control power. As a result of the proposed work, the aircraft and control system designers will be provided with a new tool capable of solving the following problem: Given the flying-quality requirements for a specified mission, obtain reduced control-surface sizes and a feedback controller that together satisfy these requirements.

The answers obtained will help reduce aircraft weight, size, and stealth and undoubtedly will result in cost savings in many current and future military and commercial aircraft design and procurement programs.

This paper is organized as follows. Section II reviews several of the more useful performance measures that can be posed as LMIs. Section III formulates the proposed integrated aircraft–controller synthesis problem as a COP. Section IV proposes a numerical algorithm for solving the COP. An example applying the proposed methodology to the case of F-14 fighter aircraft is included in Sec. IV. This example illustrates how some of the flying-quality requirements can be expressed as LMIs. Section V discusses several classes of maneuverability requirements and shows how they can be included in the COP problem formulation. The paper ends with conclusions.

II. Background: Design Requirements and LMIs

The number of control synthesis problems that can be posed as LMIs is exhaustive (e.g., Ref. 8 and references therein). In this section we review the \mathcal{H}_∞ synthesis problem and show how it can be posed as an LMI.

Consider Fig. 1. Let \mathcal{G} denote the generalized linear aircraft model. It may consist of the linear aircraft model plus the weights usually appended to this model as a part of the design process. Such weights may include gains, integrators, low-pass filters, notches, and so forth. The exogenous inputs w represent commands and disturbances acting on the aircraft. The vector z may include outputs whose size should be kept small in the presence of inputs w . Input u denotes aircraft control inputs such as elevators, ailerons, rudder, and thrust; output y denotes the signals available from the sensor suite.

Suppose the close-loop system in Fig. 1 is stable. Let $T_{zw}(\mathcal{G}, \mathcal{C})$ denote the closed-loop transfer matrix from w to z . Then, the infinity norm of $T_{zw}(\mathcal{G}, \mathcal{C})$ is defined as the supremum over all frequencies of its largest singular value:

$$\|T_{zw}(\mathcal{G}, \mathcal{C})\|_\infty := \sup_{\omega} \{\bar{\sigma}[T_{zw}(j\omega)]\}$$

where $\bar{\sigma}$ denotes the maximum singular value of $T_{zw}(j\omega)$. An

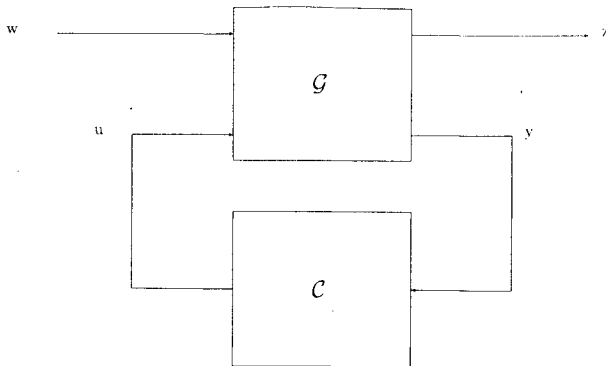


Fig. 1 Feedback interconnection of linear aircraft model \mathcal{G} and controller \mathcal{C} .

interesting physical interpretation of this quantity is that it represents the peak power gain from w to z (Ref. 11):

$$\|T_{zw}(\mathcal{G}, \mathcal{C})\|_\infty = \sup \left\{ \frac{\text{pow}(z)}{\text{pow}(w)}, \text{pow}(w) < \infty \right\}$$

where

$$\text{pow}(x) := \left[\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \|x(t)\|^2 dt \right]^{\frac{1}{2}} \quad (1)$$

Furthermore, it also represents an upper bound on the average output power, when the infinity norm of the input is bound by 1 (Ref. 11):

$$\|T_{zw}(\mathcal{G}, \mathcal{C})\|_\infty \geq \sup\{\text{pow}(z) : \|w\|_\infty \leq 1\} \quad (2)$$

These interpretations of the \mathcal{H}_∞ norm of T_{zw} will be used to formulate the gust response requirements.

The \mathcal{H}_∞ synthesis problem is to find a feedback controller \mathcal{C} that will stabilize the generalized aircraft model \mathcal{G} and will make the infinity norm of the transfer matrix T_{zw} from inputs w to outputs z less than a given number $\gamma > 0$:

$$\|T_{zw}(\mathcal{G}, \mathcal{C})\|_\infty < \gamma$$

Let \mathcal{G} admit the realization

$$\mathcal{G} = \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = Cx + Du \\ y = x \end{cases} \quad (3)$$

where $x \in \mathbb{R}^n$, $w \in \mathbb{R}^m$, $u \in \mathbb{R}^q$, and $z \in \mathbb{R}^p$. Note that in Eq. (3), we assumed that all of the states of \mathcal{G} are available for feedback. This assumption is not unreasonable for many aircraft control problems. Moreover, for the case where not all states are available for feedback, this problem can be reduced to solving a state feedback problem for an auxiliary plant^{12,13} or by adding an extra LMI constraint.^{14,15} Assume that (A, B_2) is stabilizable and that D has full column rank. Then, there exists a feedback controller \mathcal{C} such that $\|T_{zw}(\mathcal{G}, \mathcal{C})\|_\infty < \gamma$ if and only if there exist matrices $Y = Y' \in \mathbb{R}^{n \times n}$, $Y > 0$ and $W \in \mathbb{R}^{q \times n}$ such that^{8,12}

$$R(W, Y, \gamma) :=$$

$$\begin{bmatrix} AY + YA^T + B_2 W + W^T B_2^T & B_1 & (CY + DW)^T / \gamma \\ B_1^T & -I & 0 \\ (CY + DW) / \gamma & 0 & -I \end{bmatrix} < 0 \quad (4)$$

If such matrices W and Y exist, then one such controller \mathcal{C} is a constant-gain matrix K given by $K = WY^{-1}$.

Efficient interior-point numerical techniques for solving LMIs can be found in Refs. 8–10, where Y and W are expressed as affine functions of a decision vector ξ and basis matrices Y_i, W_i : $Y = Y(\xi) = Y_0 + \sum_{i=1}^s \xi_i Y_i$, and $W = W(\xi) = W_0 + \sum_{i=1}^s \xi_i W_i$, $\xi \in \mathbb{R}^s$, and $s = [n(n+1)/2] + nq$. The numerical search is then performed over the decision vector ξ . LMI-Lab employs one of the interior-point techniques, the so-called projective method of Nesterov and Nemirovskii.⁹ The readers interested in learning more about this and other interior-point algorithms are referred to the recent book by Nesterov and Nemirovskii.¹⁶

III. Problem Formulation

The general problem that we address in this paper can be stated as follows. Given aircraft mission requirements and an initial (baseline) aircraft configuration, find the minimal size of aircraft aerodynamic surfaces and a feedback controller that, together, will satisfy the mission requirements. A concise problem formulation and a possible solution are discussed next. The key idea is to relate the formulated problem to that of minimizing a linear cost (a function of aircraft parameters) subject to linear matrix constraints that represent the mission performance requirements.

Let ζ be the vector of aircraft parameters to be minimized. Since ζ consists of physical sizes of the aircraft surfaces, we constrain $\zeta_i > 0$. Let $J = c^T \zeta$ be the cost function, where $c_i > 0$ denotes the relative cost (weight) we choose to assign to each aircraft parameter ζ_i . For example, if we seek to minimize the total mass of the physical control

Table 1 α vs maximum eigenvalue (λ_{\max}) of $R(S, G, \gamma)$

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
λ_{\max}	0.0063	0.0112	0.0143	0.0155	0.0145	0.0115	0.0067	0.0004	-0.0052

surfaces and ζ_i were the physical area of each control surface, then c_i might be the mass per unit area, and J the total weight of the subject components. Or, c_i can represent cost per unit area, in which case J is the total cost of adding surfaces under consideration. The cost function $J(\zeta)$ is clearly linear in ζ .

Now suppose the matrices A , B_1 , and B_2 in Eq. (3) can be expressed as affine functions of the aircraft parameters. In other words, let

$$\begin{aligned} A &= A(\zeta) = A_0 + \sum_{i=1}^r \zeta_i A_i \\ B_1 &= B_1(\zeta) = B_{10} + \sum_{i=1}^r \zeta_i B_{1i} \\ B_2 &= B_2(\zeta) = B_{20} + \sum_{i=1}^r \zeta_i B_{2i} \end{aligned} \quad (5)$$

where $\zeta \in \mathcal{R}$ is the vector of aircraft parameters to be optimized. Examples 1 and 2 (see Secs. IV and V) demonstrate that parameters such as the area of an aerodynamic control surface occur naturally as affine variables in aircraft dynamics. To simplify notation, in the sequel we will omit writing explicitly the dependence of the matrices A , B_1 , and B_2 , on ζ and W and Y on ξ . The controller matrices (Y, W) will consistently be functions of ξ and the aircraft matrices (A, B_1, B_2) will always be functions of ζ .

Let $\gamma > 0$ be given. Define

$$\Phi(\mathcal{G}, \gamma) = \{Y, W, \zeta : R(W, Y, \gamma) < 0, Y > 0, \zeta_i > 0, \forall i = 1, r\} \quad (6)$$

where R is defined in Eq. (4). We now propose the following COP:

Minimize

$$\{J = c^T \zeta\}$$

Subject to

$$(Y, \zeta, W) \in \Phi(\mathcal{G}, \gamma) \quad (7)$$

A solution to this COP includes a state-feedback controller K that satisfies the \mathcal{H}_∞ constraint $\|T_{zw}(\mathcal{G}, K)\|_\infty < \gamma$ and a vector ζ of new optimized aircraft control-surface sizes.

It turns out that the constraint set $\Phi(\mathcal{G}, \gamma)$ is not convex, as illustrated by the following simple example. Let $\gamma = 10$ and suppose

$$\mathcal{G} = \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = Cx + Du \\ y = x \end{cases}$$

where

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \zeta_1 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \zeta_2 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \zeta_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \zeta_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B_2 = B_1, \quad C = [1 \quad 1], \quad D = 1$$

and $(\zeta_1, \zeta_2) \in (0, 1) \times (0, 1)$. Furthermore, let

$$W^1 = \begin{bmatrix} -3.3815 \\ -7.1353 \end{bmatrix}, \quad Y^1 = \begin{bmatrix} 0.7496 & 0.7566 \\ 0.7566 & 3.9542 \end{bmatrix}, \quad \zeta^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$W^2 = \begin{bmatrix} -1.8804 \\ -2.4629 \end{bmatrix}, \quad Y^2 = \begin{bmatrix} 0.4025 & 0.4053 \\ 0.4053 & 1.2627 \end{bmatrix}$$

$$\zeta^2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

Then $(Y^1, W^1, \zeta^1) \in \Phi(\mathcal{G}, 10)$ and $(Y^2, W^2, \zeta^2) \in \Phi(\mathcal{G}, 10)$, where $\Phi(\mathcal{G}, 10)$ is defined in Eq. (6).

Now let $S = \alpha Y^2 + (1 - \alpha)Y^1$, $G = \alpha W^2 + (1 - \alpha)W^1$, $\beta = \alpha \zeta^2 + (1 - \alpha)\zeta^1$, where $\alpha \in (0, 1)$. For $\Phi(\mathcal{G}, 10)$ to be convex (S, G, β) must be in $\Phi(\mathcal{G}, 10)$ for all $\alpha \in (0, 1)$. As Table 1 shows, this is not the case.

Nonconvexity of the set $\Phi(\mathcal{G}, \gamma)$ indicates that obtaining a global minimum of the COP (7) is a very difficult if not impossible task. Therefore, most numerical solutions will arrive at a local minimum, at best. One such numerical solution is discussed next.

IV. Proposed Numerical Solution

Consider the expression (4) of Sec. II. Notice that for a fixed aircraft model $(A, [B_1 \ B_2], C, D)$ the constraint $(R(W, Y, \gamma) < 0)$ is affine in W and Y . On the other hand, for a fixed controller $K = WY^{-1}$, this constraint is affine in the vector of aircraft parameters ζ . This suggests the following approach to solving the COP (7): for a fixed vector ζ , find a feasible controller K ; then, for a fixed controller K , minimize the cost function J over $\zeta \in \Phi(\mathcal{G}, \gamma)$:

1) Fix ζ

Find W, Y such that

$$\begin{bmatrix} Y & 0 \\ 0 & -R(W, Y, \gamma) \end{bmatrix} > 0 \quad (8)$$

2) For a fixed W, Y

Minimize

$$J(\text{over } \zeta)$$

Subject to

$$\begin{bmatrix} J - c^T \zeta & 0 & 0 \\ 0 & \text{diag}(\zeta_i) & 0 \\ 0 & 0 & -R(W, Y, \gamma) \end{bmatrix} > 0 \quad (9)$$

3) Go to step 1 until exit criterion is satisfied.

This algorithm involves solving two LMI optimization problems at each iteration and was implemented in MATLAB using LMI-Lab. The first step of the algorithm involves solving a so-called feasibility problem. The second step requires minimization of a linear cost function subject to an LMI constraint. LMI-Lab provides algorithms to solve each problem. Nemirovskii and Gahinet⁹ provide a detailed description of the Projective Method Algorithm used to obtain solutions. Importantly, using the Projective Method Algorithm to solve the feasibility problem does not require an initial feasible guess of W and Y . Furthermore, because initial values of control-surface sizes are always available following the preliminary aircraft control-power design, they can be used to initialize the vector ζ .

In recent work, Goh et al.¹⁷ have shown that, because each step of the proposed algorithm involves optimization of the nonsmooth function, the algorithm is not guaranteed to converge to a local minimum. Therefore, applying it to solving the COP (7) requires multiple runs using a variety of initial conditions. And, as the following example demonstrates, the algorithm converges to a local minimum for the problem at hand.

Example 1: Integrated Control Power-Controller Design for F-14 Fighter Aircraft. This example serves to illustrate the utility of the proposed methodology. In particular, we show how \mathcal{H}_∞ norm constraints can be used to formulate the following flying-quality requirements: stability, bandwidth, control-surface deflections, gust response, and closed-loop maneuverability. Because of the availability of the component stability derivative data for the F-14 aircraft, a longitudinal control problem similar to the one discussed in Ref. 18 was selected for this example. Although the F-14 is not an RSS aircraft, the problem formulation can easily incorporate unstable aircraft dynamics.

The F-14 linear model was obtained for the carrier-approach flight condition. The vehicle parameters to be optimized were the normalized control powers of both the horizontal stabilators $C_{L_{stab}}$ and the direct lift control (DLC) $C_{L_{DLC}}$. Stabilators form a moving horizontal tail of the F-14, and DLC surfaces are attached to its wings. Consequently, let the vector of aircraft parameters ζ be defined as

$$\zeta = [\zeta_1, \zeta_2]^T := \left[\frac{C_{L_{stab}}}{C_{L_{stab}(\text{nominal})}}, \frac{C_{L_{DLC}}}{C_{L_{DLC}(\text{nominal})}} \right]^T$$

The choice of optimization parameters was driven by the unusual characteristics of the DLC. This is significant because the DLC control power is neither linear in deflection nor proportional to the size of the DLC surfaces. Consequently, we must optimize a quantity whose influence on the aircraft dynamics is linear. Normalized control power therefore was chosen as the optimization parameter to adhere to the assumption that the aircraft dynamics be reasonably modeled by a linear system. In the actual aircraft implementation, the controller will have to include a nonlinear schedule on the DLC deflection to achieve linearity in the commanded control power. On the other hand, since the stabilators are a conventional aerodynamic surface, their control power is, in fact, linear in both deflection and surface area, and any one of the related parameters (tail volume, surface area, or absolute control power) could have been selected. For consistency, normalized stabilator control power was chosen.

Initially, the cost-function weights on the aircraft parameters were arbitrarily chosen to be $c = [3 \ 1]^T$. We remind the reader that these weights can represent normalized cost in dollars, weight in pounds, and so forth. The reference input of interest was commanded flight-path angle γ_c , and the outputs to be regulated were the actuator deflections, the angle-of-attack error, and the flight-path-angle error. The disturbance input was a vertical gust. The control inputs were stabilators and DLC deflection (thrust was assumed constant), and the full state vector was assumed to be available for feedback. Thus, the problem can be stated as follows:

Find the aircraft parameters ζ and a state-feedback controller $K = WY^{-1}$ that minimize the total cost $J = c^T \zeta$ subject to the following requirements:

1) Closed-loop stability. Resulting closed-loop system must be stable.

2) Step response. Controller must track a step flight-path-angle command, γ_{cmd} , with no steady-state error in angle of attack α or flight-path angle γ .

3) Closed-loop performance. In the presence of a vertical gust disturbance, w_{gust} , with a magnitude of 5 ft/s, the stabilator deflection should not exceed 20 deg, the DLC deflection should not exceed 40 deg, and the angle-of-attack error should not exceed 1.5 deg (all quantities are root mean square).

4) Closed-loop maneuverability. Aircraft must track a flight-path angle ($\gamma = \theta - \alpha$) of 3 deg with the DLC and stabilator deflected no more than 40 and 20 deg, respectively.

These requirements will be satisfied if the following \mathcal{H}_∞ constraints are met:

1) The step response requirement implies

$$\|T_{z_1 \gamma_{cmd}}(G, C)\|_\infty < \infty, \quad \text{where} \quad z_1 := \left[\frac{\alpha_{error}}{s}, \frac{\gamma_{error}}{s} \right]^T$$

2) The closed-loop performance implies

$$\|T_{z_2 w_{gust}}(G, C)\|_\infty < \frac{1}{5 \text{ ft/s}}$$

where

$$z_2 := \left[\frac{\delta_{stab}}{20 \text{ deg}}, \frac{\delta_{DLC}}{40 \text{ deg}}, \frac{\alpha}{1.5 \text{ deg}} \right]^T$$

3) The closed-loop maneuverability implies

$$\|T_{z_3 \gamma_{cmd}}(G, C)\|_\infty < \frac{1}{3 \text{ deg}}, \quad \text{where} \quad z_3 := \left[\frac{\delta_{stab}}{20 \text{ deg}}, \frac{\delta_{DLC}}{40 \text{ deg}} \right]^T$$

A sufficient condition, albeit conservative, to meet all three constraints is $\|T_{zw}(G, C)\|_\infty < 1$, where

$$w := [w_{gust}, \gamma_{cmd}]^T = [5 \text{ ft/s}, 3 \text{ deg}]^T$$

$$z := \left[\frac{\delta_{stab}}{20 \text{ deg}}, \frac{\delta_{DLC}}{40 \text{ deg}}, \frac{\alpha}{1.5 \text{ deg}}, \frac{\alpha_{error}}{c_1 s}, \frac{\gamma_{error}}{c_2 s} \right]^T \quad (10)$$

and c_1 and c_2 are any finite positive numbers. These constraints lead to the formulation of the synthesis model discussed next.

To proceed with the problem description we need to define the following terms:

$C_{X_i} := \partial X / \partial y$ nondimensional trimmed force-moment coefficient, where X can be lift (L), drag (D), or pitching moment (M)

$C_{X_y} := \partial X / \partial y$ nondimensional stability derivative, where $y \neq t$ is a nondimensional state or control deflection

$\bar{c} :=$ wing mean chord

$I_{yy} :=$ moment of inertia about the lateral (y) axis

$l_{stab} := (C_{M_{stab}} / C_{L_{stab}})$ stabilator lever arm (distance from CG to aerodynamic center of the stabilator)

$m :=$ mass

$Q :=$ dynamic pressure

$q :=$ pitch rate about body y axis

$S :=$ wing reference area

$U :=$ aircraft velocity resolved in the body x axis

$V :=$ aircraft velocity

$\alpha :=$ angle of attack

$\sigma_\alpha := \sigma_{w(\text{turbulence})} / V$ turbulence variance

$\sigma_\gamma :=$ maximum flight-path-angle command amplitude

$\theta :=$ pitch attitude

Next, the aircraft stability derivatives were expressed as the sum of their wing-body (wb), stabilator ($stab$), and DLC contributions¹⁹:

$$C_{D_{\alpha(wb)}} := C_{D_{\alpha}}, \quad C_{L_{\alpha(wb)}} := C_{L_{\alpha}} - C_{L_{stab}}$$

$$C_{M_{\alpha(wb)}} := C_{M_{\alpha}} + l_{stab} C_{L_{stab}}, \quad C_{L_{q(wb)}} := C_{L_q} - 2l_{stab} C_{L_{stab}}$$

$$C_{M_{q(wb)}} := C_{M_q} + 2l_{stab}^2 C_{L_{stab}}, \quad C_{L_{\dot{\alpha}(wb)}} := C_{L_{\dot{\alpha}}} - 2l_{stab} C_{L_{stab}}$$

$$C_{M_{\dot{\alpha}(wb)}} := C_{M_{\dot{\alpha}}} + 2l_{stab}^2 C_{L_{stab}}$$

The linear aerodynamic model of F-14 is derived next, followed by an outline of the way it was used to form a synthesis model. The states x_{aero} of the basic longitudinal linear aircraft model are $x_{aero} = [U/V \ \alpha \ q \ \theta]^T$. Let T be a rotation-scaling matrix:

$$T := QS \begin{bmatrix} -\cos(\alpha_0) & \sin(\alpha_0) & 0 \\ -\sin(\alpha_0) & -\cos(\alpha_0) & 0 \\ 0 & 0 & \bar{c} \end{bmatrix}$$

where α_0 is the trim value of α . Let I_m represent the inertia- $\dot{\alpha}$ matrix:

$$I_m = I_m(\zeta) = I_{m0} + \zeta_1 I_{m1}$$

$$= \begin{bmatrix} mV & & & \\ & mV & & \\ & & I_{yy} & \\ & & & 1 \end{bmatrix} - \begin{bmatrix} \frac{\bar{c}}{2V} T \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{L_{\dot{\alpha}}} & 0 \\ 0 & C_{M_{\dot{\alpha}}} & 0 \end{bmatrix} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} mV & & & \\ & mV & & \\ & & I_{yy} & \\ & & & 1 \end{bmatrix} - \begin{bmatrix} \frac{\bar{c}}{2V} T \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_{L_{\dot{\alpha}(wb)}} & 0 \\ 0 & C_{M_{\dot{\alpha}(wb)}} & 0 \end{bmatrix} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} \frac{\bar{c}}{2V} T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2l_{stab} C_{L_{stab}(\text{nominal})} & 0 \\ 0 & -2l_{stab}^2 C_{L_{stab}(\text{nominal})} & 0 \end{bmatrix} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \zeta_1$$

Now, the state matrix of the aircraft has the following form:

$$A_{\text{aero}} = A_{\text{aero}_0} + A_{\text{aero}_1} \zeta_1 + A_{\text{aero}_2} \zeta_2 = I_m^{-1} \left[\begin{pmatrix} T \begin{bmatrix} 2C_{D_t} + C_{D_u} V^{-1} & C_{D_{\alpha(wb)}} - C_{L_t} & 0 \\ 2C_{L_t} + C_{L_u} V^{-1} & C_{L_{\alpha(wb)}} + C_{D_t} & C_{L_{q(wb)}} (\bar{c}/2V) \\ 2C_{M_t} + C_{M_u} V^{-1} & C_{M_{\alpha(wb)}} & C_{M_{q(wb)}} (\bar{c}/2V) \end{bmatrix} + \begin{bmatrix} -mg \cos(\theta_0) \\ -mg \sin(\theta_0) \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & -mV \sin(\alpha_0) \\ 0 & 0 & mV \cos(\alpha_0) \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{pmatrix} \right] \\ + I_m^{-1} \left[\begin{bmatrix} C_{L_{\text{stab(nominal)}}} T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2\frac{\bar{c}l_{\text{stab}}}{2V} \\ 0 & l_{\text{stab}} & 2\frac{\bar{c}l_{\text{stab}}}{2V} \\ 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \zeta_1 \right] \quad (11)$$

$$B_{\text{aero}} = B_{\text{aero}_1} \zeta_1 + B_{\text{aero}_2} \zeta_2 + I_m^{-1} \left[\begin{bmatrix} C_{L_{\text{stab(nominal)}}} T \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ -l_{\text{stab}} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \zeta_1 + I_m^{-1} \left[\begin{bmatrix} C_{L_{\text{DLC(nominal)}}} T \begin{bmatrix} 0 & C_{D_{\text{DLC}}}/C_{L_{\text{DLC(nominal)}}} \\ 0 & 1 \\ 0 & C_{M_{\text{DLC}}}/C_{L_{\text{DLC(nominal)}}} \\ 0 & 0 \end{bmatrix} \end{bmatrix} \zeta_2 \right] \quad (12)$$

Note that these matrices include dynamic coupling and gravity terms, as well as the aerodynamic forces. This model was verified by comparison with the nominal-state matrices ($\zeta = [1, 1]^T$) obtained from the linearization of the nonlinear model described in Ref. 18. Furthermore, observe that the inertia- α matrix I_m is a function of ζ_1 and, therefore, aircraft-state and input matrices depend nonlinearly on ζ_1 . However, the impact of reduced ζ_1 on I_m was negligible. Therefore, I_m evaluated at $\zeta_1 = 1$ was used to compute A_{aero} and B_{aero} .

Now, using the aerodynamic data defined above, a synthesis model \mathcal{G} has the following form:

$$\mathcal{G} = \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = Cx + Du \end{cases} \quad (13)$$

where w and z are defined in Eq. (10), and

$$A = \begin{bmatrix} & & 0 & 0 \\ & A_{\text{aero}_0} & & \\ & & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} & & 0 & 0 \\ & A_{\text{aero}_1} & & \\ & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \zeta_1 \\ B_1 = \begin{bmatrix} & 0 \\ \sigma_{\alpha} A_{\text{aero}_0}(:, 2) & 0 \\ & 0 \\ 0 & 0 \\ 0 & \sigma_{\gamma} \end{bmatrix} + \begin{bmatrix} & 0 \\ \sigma_{\alpha} A_{\text{aero}_1}(:, 2) & 0 \\ & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \zeta_1 \\ B_2 = \begin{bmatrix} B_{\text{aero}_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \zeta_1 + \begin{bmatrix} B_{\text{aero}_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \zeta_2 \\ C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}, \quad D = \begin{bmatrix} 1/20 & 0 \\ 0 & 1/40 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (14)$$

Note that the second column of A shows up in B_1 because the aircraft sees a sharp-edged perturbation in the vertical air mass (W) as a perturbation in angle of attack ($\alpha = W/V$). [Here $A(:, 2)$ represents the second column of A .] Moreover, the weights c_1 and c_2 on the outputs α_{error}/s and γ_{error}/s were set to 10 to ensure that the errors in each loop go to zero reasonably fast.

To summarize, the COP to be solved in this example can be stated as follows. Let the generalized linear aircraft model be given by Eqs. (13) and (14) and let $\zeta = [\zeta_1 \ \zeta_2]^T$. Set $\gamma = 1$, $c = [3 \ 1]^T$, and Minimize

$$\{J = c^T \zeta\}$$

Subject to

$$(Y, W, \zeta) \in \Phi(\mathcal{G}, \gamma) \quad (15)$$

where

$$\Phi(\mathcal{G}, \gamma) = \{Y, W, \zeta : R(W, Y, \gamma) < 0, Y > 0, \zeta_i > 0, i = 1, 2\} \quad (16)$$

The algorithm introduced in Sec. IV was used to obtain a numerical solution to this problem. To test the convergence properties

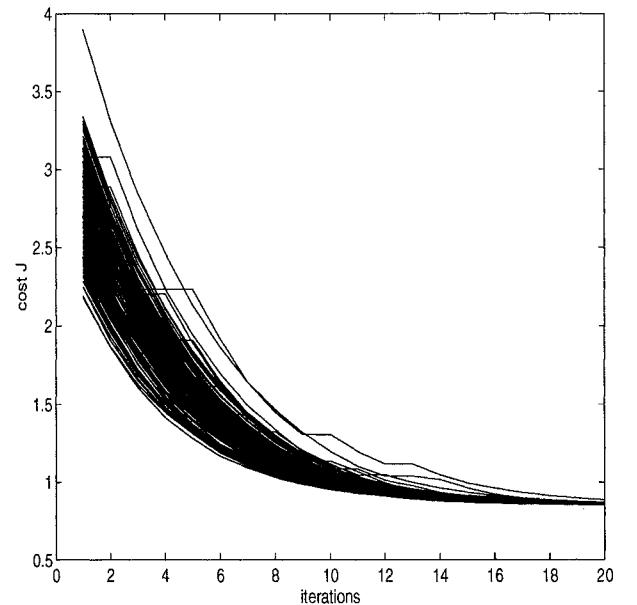


Fig. 2 Optimization history for the cost J (example 1).

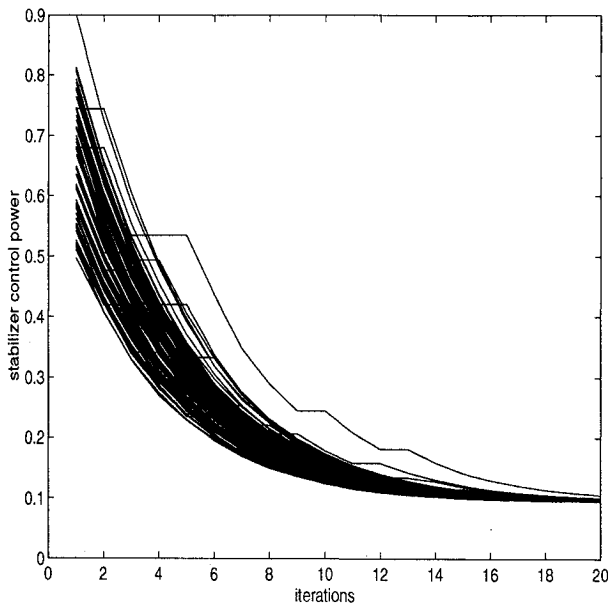


Fig. 3 Optimization history for the stabilator control power ζ_1 (example 1).

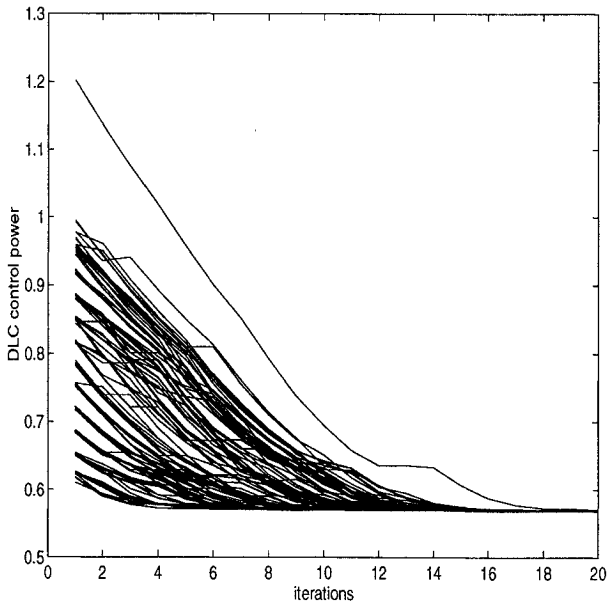


Fig. 4 Optimization history for the size DLC control power ζ_2 (example 1).

of this algorithm the initial values for plant parameters ζ_0 were obtained from the set $\{\zeta_0 = [(\zeta_{10}, \zeta_{20})]' : (\zeta_{10}, \zeta_{20}) \in [0.6, 1] \times [0.6, 1]\}$. From this set, 100 initial points were selected.

Figure 2 shows the optimization history for the cost J . Clearly, it converges to the same final value for all initial conditions: $J_{\text{final}} = 0.856$. Figures 3 and 4, respectively, contain the optimization history for the stabilator and DLC parameters ζ_1 and ζ_2 . Again, both ζ_1 and ζ_2 converge to the same final value for all initial conditions: $\zeta_{1\text{final}} = 0.096$ and $\zeta_{2\text{final}} = 0.568$. These data suggest that the algorithm had converged to a local minimum. More important, the final values of ζ_1 and ζ_2 (whether minimum or not) were considerably smaller than the values (1,1) of ζ_1 and ζ_2 for the initial aircraft configuration. This indicates that for the design requirements at hand, characterized by the \mathcal{H}_∞ norm of a closed-loop transfer matrix $T_{zw}(\mathcal{G}, K)$, where $K = WY^{-1}$, the control-surface sizes can be reduced without sacrificing performance.

Now, a natural question is whether the reduction in the control-surface sizes resulted in the increase of the required closed-loop system bandwidth. Consider Fig. 5, which depicts the optimization history of the \mathcal{H}_∞ norm of $T_{zw}(\mathcal{G}, K) = (C + D * K)(sI - A - B_2 * K)^{-1} B_1$ for each initial value of ζ . Recall that the sufficient condition for the design requirements to be satisfied is $\|T_{zw}(\mathcal{G}, K)\|_\infty < 1$. Predictably, as the control-surface sizes decrease, the \mathcal{H}_∞ norm of $T_{zw}(\mathcal{G}, K)$ approaches 1. This trend suggests that the local minimum obtained by the algorithm occurs on the boundary of the constraint set $\Phi(\mathcal{G}, \gamma)$ and that the decrease in the control-surface sizes was achieved at the expense of increasing the bandwidth of $T_{zw}(\mathcal{G}, K)$. This conclusion is corroborated by Fig. 6, where the maximum singular value of $T_{zw}(\mathcal{G}, K)\{\bar{\sigma}[T_{zw}(\mathcal{G}, K)]\}$ is plotted vs frequency. Here, each plot of $\bar{\sigma}[T_{zw}(\mathcal{G}, K)]$ is computed for the values of $K = WY^{-1}$ and ζ obtained from the first run of the optimization algorithm initialized with $\zeta = [1, 1]^T$ (see Figs. 2–4). Clearly, the bandwidth of the $\bar{\sigma}[T_{zw}(\mathcal{G}, K)]$ increases as ζ_1, ζ_2 decrease.

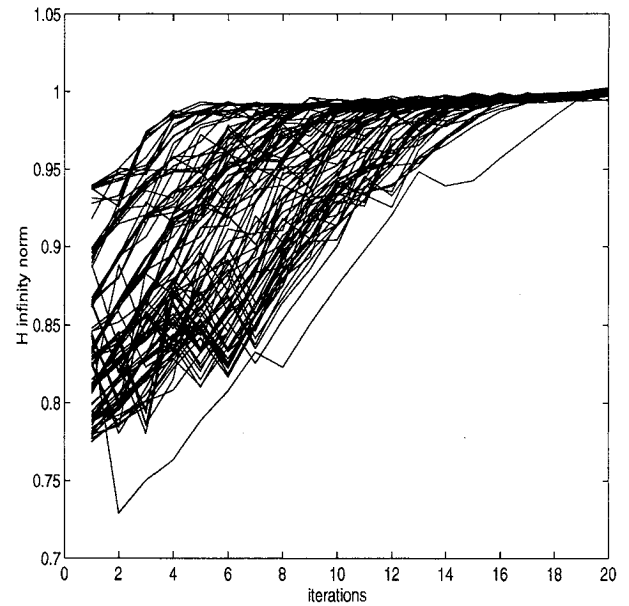


Fig. 5 Optimization history for the $\|T_{zw}(\mathcal{G}, K)\|_\infty$ (example 1).

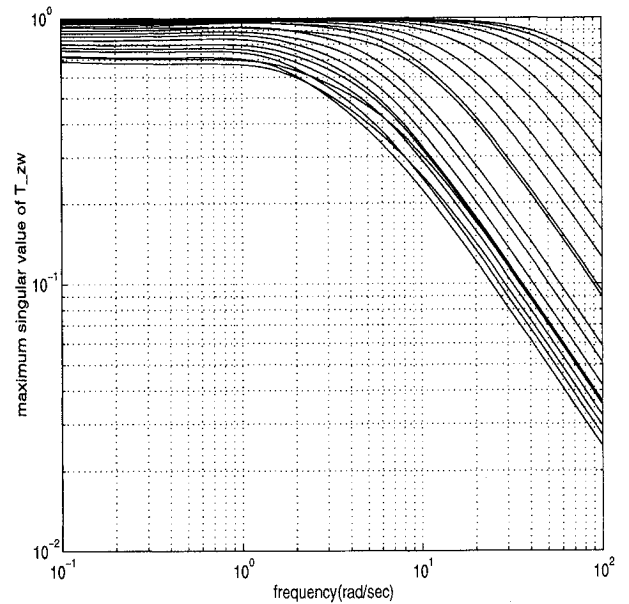


Fig. 6 Optimization history of $\bar{\sigma}[T_{zw}(\mathcal{G}, K)]$ (example 1).

Now, one should expect that increasing the bandwidth of the $T_{zw}(\mathcal{G}, K)$ resulted in increasing the stabilator and DLC control-loop bandwidths. These can be obtained by analyzing the Bode plots of the diagonal elements of the transfer matrix $T_{\text{control}} = K(sI - A - B_2 * K)^{-1} B_2$. Figures 7 and 8 show the Bode plots for the stabilator and DLC control loops, respectively. The stabilator control loop was clearly the one that had its bandwidth increased considerably to compensate for the reduction in size of the stabilator. However, the largest bandwidth of the plots in Fig. 7 is less than 40 rad/s. This does

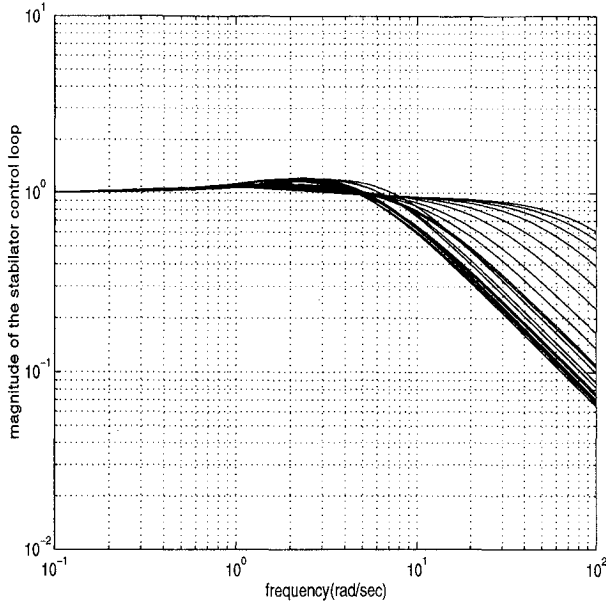


Fig. 7 Optimization history for the stabilator control loop (example 1).

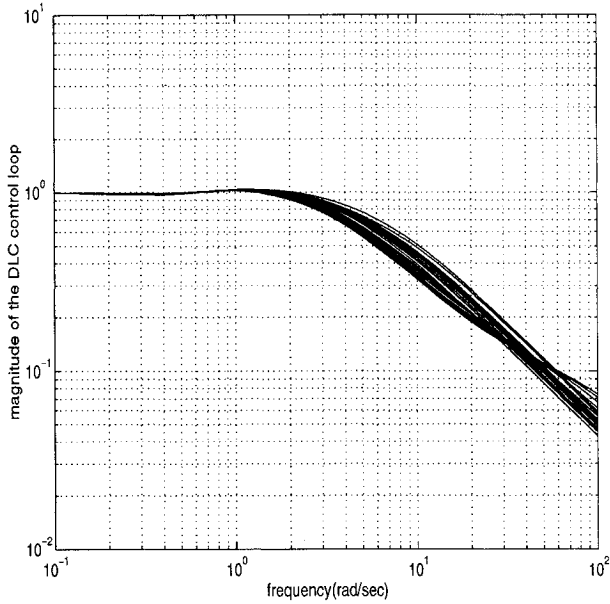


Fig. 8 Optimization history for the DLC control loop (example 1).

not exceed 50 rad/s, a typical bandwidth of the F-14 flight-control actuators.

V. Including Maneuverability Requirements

The factors that determine the size of aircraft control surfaces are not limited to the closed-loop performance requirements discussed in Sec. IV. For example, in the case of aircraft directional dynamics, the size of the vertical tail is not usually determined by a closed-loop performance requirement such as gust response, but by the loss of thrust in one of the engines. This is known as an engine-out condition. If it occurs, the rudder must be large enough to counteract the adverse yawing moment generated by the thrust in the remaining engine. Numerous other factors, known as open-loop maneuverability requirements, must be included in the integrated aircraft control power-controller design problem. In this section, we discuss the open-loop maneuverability requirements and show how some of them can be included in the integrated control power-controller design problem.

Consider an open-loop system in Fig. 9. Let \mathcal{G}_{nl} denote the nonlinear aircraft dynamics. A typical open-loop maneuverability specification for such a system has the following form: Given a maximum

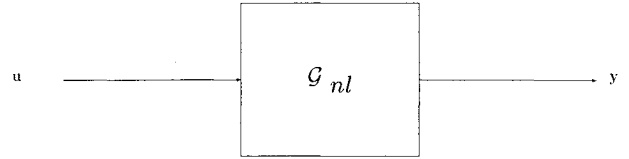


Fig. 9 Open-loop formulation for maneuverability constraints.

control input $u = u_{\max}$, the steady-state response of a scalar output y must exceed a certain threshold r_{thres} .

Open-loop maneuverability specifications fall into two general classes: dynamic and static. A typical open-loop static maneuverability requirement is for the aircraft to have sufficient control power to maintain a given torque in a pitch, roll, or yaw axis with all control effectors fully deflected or deflected no further than a given limit. On the other hand, a typical open-loop dynamic maneuverability requirement is to guarantee a given angular rate at full deflection of all control effectors at the same or different flight condition. Next, we show how a typical open-loop dynamic maneuverability requirement can be included in the F-14 example.

Example 2: Including Open-Loop Maneuverability Requirements in the Integrated Plant-Controller Design Problem for F-14. Consider a maximum pitch rate requirement at the F-14 approach flight condition. Since the required pitch rate is constant, the pitch acceleration \dot{q} must be zero at a maximum stabilator deflection. Using the pitching moment equation at this flight condition, we obtain

$$\begin{aligned} I_{yy}\dot{q} &= QSc(C_{M_0} + C_{M_\alpha}\alpha_0 + C_{M_q}q_{\max} + C_{L_{\text{stab}}}\text{stab}_{\max}) \\ -T_0p_y &= 0 = QSc[C_{M_0} + (C_{M_{\alpha(wb)}} - l_{\text{stab}}C_{L_{\text{stab}}})\alpha_0 \\ &\quad + (C_{M_{q(wb)}} - 2l_{\text{stab}}^2C_{L_{\text{stab}}})q_{\max} + l_{\text{stab}}C_{L_{\text{stab}}}\text{stab}_{\max}] \\ &\quad - Y_0p_y \end{aligned} \quad (17)$$

where C_{M_0} is the nominal aircraft pitching moment, T_0 is the trim thrust setting, p_y is the engine's moment arm, q_{\max} is the maximum pitch-rate requirement, and stab_{\max} represents a maximum stabilator deflection in the appropriate direction. Equation (17) is affine in $C_{L_{\text{stab}}}$. To ensure sufficient stabilator authority to provide q_{\max} the pitch-rate maneuverability requirement can be posed as the following LMI:

$$k_1 + k_2\zeta_1 > 0 \quad (18)$$

where

$$k_1 = QSc(C_{M_0} + C_{M_{\alpha(wb)}}\alpha_0 + C_{M_{q(wb)}}q_{\max}) - T_0p_y$$

$$\begin{aligned} k_2 &= QSc(-l_{\text{stab}}C_{L_{\text{stab(nominal)}}}\alpha_0 - 2l_{\text{stab}}^2C_{L_{\text{stab(nominal)}}}q_{\max} \\ &\quad + l_{\text{stab}}C_{L_{\text{stab(nominal)}}}\text{stab}_{\max}) \end{aligned}$$

Similar arguments can be used to obtain constraints on other angular rates. Constraint (18) now can be included in the F-14 aircraft-controller optimization example:

Minimize

$$\{J = c^T \zeta\}$$

Subject to

$$(Y, \zeta, W) \in \Phi_1(\mathcal{G}, \gamma)$$

where

$$\Phi_1(\mathcal{G}, \gamma) = \{Y, W, \zeta : R(W, Y, \gamma) < 0,$$

$$k_1 + k_2\zeta_1 > 0, Y > 0, \zeta_i > 0, \forall i = 1, 2\} \quad (19)$$

where R is defined in Eq. (4). Here, as in the previous example, γ was set to 1 and c to $[3 \ 1]^T$.

This problem can be solved by adding the constraint $k_1 + k_2\zeta_1 > 0$ to the algorithm introduced in Sec. IV:

1) Fix ζ

Find W, Y such that

$$\begin{bmatrix} Y(\xi) & 0 \\ 0 & -R(W, Y, \gamma) \end{bmatrix} > 0 \quad (20)$$

2) For a fixed W, Y
Minimize

$$J(\text{over } \zeta)$$

Subject to

$$\begin{bmatrix} J - c^T \zeta & 0 & 0 & 0 \\ 0 & \text{diag}(\zeta_i) & 0 & 0 \\ 0 & 0 & -R(W, Y, \gamma) & 0 \\ 0 & 0 & 0 & k_1 + k_2 \zeta_1 \end{bmatrix} > 0 \quad (21)$$

3) Go to step 1 until exit criterion is satisfied.

In this example q_{\max} was set to 20 deg/s and stab_{\max} was set to 20 deg. As in example 1, the algorithm was executed using different initial values of ζ . It converged to the same final values of the cost J and the vector of aircraft parameters $\zeta = [\zeta_1 \ \zeta_2]^T$. The final values for J , ζ_1 , and ζ_2 were 2.0043, 0.4776, and 0.5714, respectively. Note that the final value of ζ_1 in this case is considerably higher than the final value of ζ_1 in example 1. Clearly, this is a result of the pitch-rate maneuverability requirement added in this example.

VI. Conclusions

In this paper, we considered the problem of integrated aircraft control power-feedback controller design. The key contribution of the paper was to formulate this problem as a COP where the cost to be optimized is a linear functional of the aircraft control-power parameters and the constraint set is defined using LMIs. It was shown that the constraint set is not convex, and a numerical solution was proposed. Two applications of the resulting methodology to the problem of optimizing control-power parameters for the F-14 fighter aircraft were presented in examples 1 and 2. These examples have demonstrated the utility of the proposed methodology.

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References

- ¹Hyland, D. C., Junkins, J. L., and Longman, R. W., "Active Control Technology for Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3, 1993, pp. 801–821.
- ²Kumar, M. P. (ed.), *Structural Optimization: Status and Promise*, Vol. 150, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1993.
- ³Junkins, J. L., (ed.), *Mechanics and Control of Large Flexible Structures*, Vol. 129, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1990, Chap. 3.
- ⁴Jameson, A., "Aerodynamic Design via Control Theory," *Journal of Scientific Computing*, Vol. 3, 1988, pp. 233–260.
- ⁵Reuther, J., and Jameson, A., "Control Theory Based Airfoil Design for Potential Flow and a Finite Volume Discretization," *Proceedings of 32nd AIAA Aerospace Sciences Meeting and Exhibit* (Reno, NV), AIAA, Washington, DC, 1994 (AIAA Paper 94-0499).
- ⁶Lapins, M., Martorella, R. P., Klein, R. W., Meyer, R. C., and Strum, M. J., "Control Definition Study for Advanced Vehicles," NASA CR 3738, 1983.
- ⁷Military Specifications Document, "Flying Qualities of Piloted Airplanes," MIL-F-8785, U.S. Government Printing Office, Washington, DC, Nov. 1980.
- ⁸Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V., "Linear Matrix Inequalities in Systems and Control Theory," *Studies in Applied Mathematics*, Vol. 15, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1994.
- ⁹Nemirovskii, A., and Gahinet, P., "The Projective Method for solving Linear Matrix Inequalities," *Proceedings of American Control Conference*, Baltimore, MD, 1994, pp. 840–844.
- ¹⁰Gahinet, P., and Nemirovskii, A., "General Purpose LMI Solves with Benchmarks," *Proceedings of 32nd IEEE Conference on Decision and Control* (San Antonio, TX), Inst. of Electrical and Electronics Engineers, 1993, pp. 3162–3165.
- ¹¹Doyle, J. C., Francis, B. A., and Tannenbaum, A. R., *Feedback Control Theory*, Macmillan, New York, 1991.
- ¹²Khargonekar, P. P., and Rotea, M. A., "Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Control: A Convex Optimization Approach," *IEEE Transactions on Automatic Control*, AC-36(7), 1991, pp. 824–837.
- ¹³Stoorvogel, A., *The \mathcal{H}_∞ Control Problem*, Prentice-Hall, Englewood Cliffs, NJ, 1992.
- ¹⁴Gahinet, P., and Apkarian, P., "An LMI-based Parametrization of all \mathcal{H}_∞ Controllers with Applications," *Proceedings of 32nd IEEE Conference on Decision and Control* (San Antonio, TX), Inst. of Electrical and Electronics Engineers, 1993, pp. 656–661.
- ¹⁵Packard, A., "Gain-Scheduling via Linear Fractional Transformations," *Systems and Control Letters*, Vol. 22, 1994, pp. 79–94.
- ¹⁶Nesterov, Y., and Nemirovskii, A., "Interior Point Polynomial Methods in Convex Programming," *Studies in Applied Mathematics*, Vol. 13, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1994.
- ¹⁷Goh, K. C., Turan, L., Safonov, M. G., Papavassilopoulos, G. P., and Ly, J. H., "Biaffine Matrix Inequality Properties and Computational Methods," *Proceedings of American Control Conference*, Baltimore, MD, 1994, pp. 850–855.
- ¹⁸Niewoehner, R. J., and Kammer, I. I., "Design of an Autoland Controller for a Carrier Based F-14 Aircraft Using \mathcal{H}_∞ output Feedback Synthesis," *Proceedings of American Control Conference*, Baltimore, MD, 1994, pp. 2501–2505.
- ¹⁹Hoak, D. E., and Carlson, J. W., "USAF Stability and Control Handbook," Douglas Aircraft Co., Contract AF 33(616)-6460, Project 8219, Task 82160, April 1960.